Line Element Based 2-D Magnetic and Electric Force Computation for the Problems with Touched Objects

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Abstract — With virtual work method, the magnetic force can be precisely computed by integrating around one layer of elements surrounding the objects. However, when the objects contact with others, it becomes impossible to get the integration along such layer. In this paper, a line element based virtual work approach is presented to compute the force of 2-dimensional magnetic field, eddy-current field and electric field when the objects contact with others. General and unified formulations are deduced and the implementation methodology is presented. The effectiveness of additional line elements to the accuracy of the solution is pointed out. The accuracy of the proposed method is verified by the results with analytical methods.

I. INTRODUCTION

Finite element method (FEM) has been widely used in electromagnetic field computation for performance analysis. Magnetic and electric force is one important performance to be computed with high accuracy. The virtual work method can be used to compute the magnetic force due to its high accuracy and only one filed solution is needed to compute [1-2]. With virtual work method, the force on objects is computed by integrating around one layer of elements surrounding the objects. When the objects contact with others, for example, the PMs in an electric motor which are mounted on the iron core, line elements along the surfaces of the PMs are used for the integration in force computation.

In this paper, based on line element method, a detailed and effective approach using virtual work method to compute the force of magnetic field, eddy-current field and electric field on objects touching other objects is presented. It also makes it possible that in dynamic mechanical motion problems, the moving objects can touch stationary objects as the force on the moving objects can still be precisely computed. The accuracy of the proposed method is verified by the results with analytical methods.

II. METHODS

A. Force Computation of Magnetic Field

In the rectangular element, the bottom and the top edges are parallel and are numbered by e_1 and e_2, respectively. The thickness of the rectangular is denoted by δ. In the limit as δ approaches zero, this rectangular element becomes a line element.

In order to compute the magnetic force on the object which touches another object, the line element is used for the force integration along the touched surface. The magnetic filed H can be expressed with edge elements as:

$$\vec{H} = \begin{bmatrix} \vec{w}_1 & \vec{w}_2 & \vec{w}_3 & \vec{w}_4 \end{bmatrix} \begin{bmatrix} l_1 H_1 \\ l_2 H_2 \\ l_3 H_3 \\ l_4 H_4 \end{bmatrix} = \vec{w} H \tag{1}$$

where $\vec{w} = \begin{bmatrix} \vec{w}_1 & \vec{w}_2 & \vec{w}_3 & \vec{w}_4 \end{bmatrix}$ is the shape function of the edge element in (x, y) co-ordinates.

The permeability of the line elements should be equal to that of the background, usually $\mu = \mu_0$. The magnetic coenergy in an element is written as:

$$\begin{aligned} W^* &= l_z H^T \left(\int_{\Omega_e} \frac{\mu}{2} \vec{w}^T \cdot \vec{w} \mathrm{d}\Omega \right) H \\ &= \frac{\mu l_z}{2} \begin{bmatrix} l_1 H_1 & l_2 H_2 & l_3 H_3 & l_4 H_4 \end{bmatrix} \\ &\int_{\Omega_e} \left(\begin{bmatrix} \vec{w}_1^T \cdot \vec{w}_1 & \vec{w}_1^T \cdot \vec{w}_2 & \vec{w}_1^T \cdot \vec{w}_3 & \vec{w}_1^T \cdot \vec{w}_4 \\ \vec{w}_2^T \cdot \vec{w}_1 & \vec{w}_2^T \cdot \vec{w}_2 & \vec{w}_2^T \cdot \vec{w}_3 & \vec{w}_2^T \cdot \vec{w}_4 \\ \vec{w}_3^T \cdot \vec{w}_1 & \vec{w}_3^T \cdot \vec{w}_2 & \vec{w}_3^T \cdot \vec{w}_3 & \vec{w}_3^T \cdot \vec{w}_4 \\ \vec{w}_4^T \cdot \vec{w}_1 & \vec{w}_4^T \cdot \vec{w}_2 & \vec{w}_4^T \cdot \vec{w}_3 & \vec{w}_4^T \cdot \vec{w}_4 \end{bmatrix} \right) \mathrm{d}\Omega \begin{bmatrix} l_1 H_1 \\ l_2 H_2 \\ l_3 H_3 \\ l_4 H_4 \end{bmatrix} \\ &= \frac{\mu l_z}{2} \sum_{i=1}^{4} \sum_{j=1}^{4} H_i \left(m_{ij} \right) H_j \end{aligned} \tag{2}$$

where l_z is the model depth in z direction, and

$$m_{ij} = l_i l_j \int_{\Omega_e} \vec{w}_i^T \cdot \vec{w}_j \mathrm{d}\Omega \tag{3}$$

The derivative of the coenergy with respect to displacement s gives the contribution of one line element to the nodal force,

$$F = \frac{\partial W^*}{\partial s} = \frac{\mu l_z}{2} \sum_{i=1}^{4} \sum_{j=1}^{4} l_i H_i \left(\frac{\partial m_{ij}}{\partial s} \right) l_j H_j \tag{4}$$

where

$$m_{ij} = \int_{\Omega_e} \vec{w}_i^T \cdot \vec{w}_j \mathrm{d}\Omega = \int_{\hat{\Omega}_e} \hat{\vec{w}}_i^T J^{-T} \cdot J^{-1} \hat{\vec{w}}_j |J| \mathrm{d}\hat{\Omega} \tag{5}$$

Only the Jacobin matrix depends on the displacement; the functions $\hat{\vec{w}}_i$ as well as the area $\hat{\Omega}_e$ of the reference element are independent of it.

$$\begin{aligned} \frac{\partial m_{ij}}{\partial s} &= \int_{\hat{\Omega}_e} \left[\hat{\vec{w}}_i^T \frac{\partial \left(J^{-T} \cdot J^{-1} \right)}{\partial s} \hat{\vec{w}}_j |J| + \hat{\vec{w}}_i^T J^{-T} \cdot J^{-1} \hat{\vec{w}}_j \frac{\partial |J|}{\partial s} \right] \mathrm{d}\hat{\Omega} \\ &= \int_{\hat{\Omega}_e} \left[-\hat{\vec{w}}_i^T J^{-T} \frac{\partial J^T}{\partial s} J^{-T} J^{-1} \hat{\vec{w}}_j |J| \right] \mathrm{d}\hat{\Omega} + \int_{\hat{\Omega}_e} \left[-\hat{\vec{w}}_i^T J^{-T} J^{-1} \frac{\partial J}{\partial s} J^{-1} \hat{\vec{w}}_j |J| \right] \mathrm{d}\hat{\Omega} \\ &\quad + \int_{\hat{\Omega}_e} \left[\hat{\vec{w}}_i^T J^{-T} \cdot J^{-1} \hat{\vec{w}}_j \frac{\partial |J|}{\partial s} \right] \mathrm{d}\hat{\Omega} \end{aligned} \tag{6}$$

B. Force Computation of Eddy-current Field

In eddy-current field the field equations are solved in frequency domain. The force has average component and a.c. component, which are computed based on the average of coenergy and the ac fluctuation component of coenergy. The average of coenergy can be deduced as:

$$W_{av}^* = l_z \int_{\Omega} \frac{1}{4} \mathrm{Re} \left[\dot{\boldsymbol{B}} \cdot \overset{*}{\boldsymbol{H}} \right] d\Omega \tag{7}$$

The ac fluctuation component of coenergy can be expressed as:

$$\dot{W}_{ac}^{*}=l_z\int_{\Omega}\frac{1}{4}\left[\dot{\boldsymbol{B}}\cdot\dot{\boldsymbol{H}}\right]d\Omega \tag{8}$$

Therefore the average force can be computed:

$$F_{av}=\frac{\partial W_{av}^{*}}{\partial s} \tag{9}$$

And the ac fluctuation component of force in complex can be computed:

$$\dot{F}_{ac}=\frac{\partial \dot{W}_{ac}^{*}}{\partial s} \tag{10}$$

C. Force Computation of Electric Field

The coenergy of electric field is:

$$W^{*}=l_z\int_{\Omega}\frac{1}{2}\boldsymbol{D}\cdot\boldsymbol{E}d\Omega \tag{11}$$

Therefore, if replacing B by D, and H by E, the formulation of magnetic field force computation can still be used.

D. Implementation

In developed computer program, a base class is created to realize the basic algorithm. To deal with specific problems of different fields, classes for magnetic field force, electric field force and eddy-current field force are derived from the based class, respectively. Two flags on the vertexes are set up, which are defined as:

n_f = 1: the vertex is on the objects for force computation (force objects);
0: any other vertexes.
n_t = -1: the vertex on the outside edges of the force objects;
1: the vertex on the touching edges;
0: in the moving objects and in any other places.

In each triangle element, $e_f=\sum_{i=1}^{3}n_{f(i)}$, $e_t=\sum_{i=1}^{3}n_{t(i)}\Big|_{\text{only if } n_{t(i)}>0}$. If $e_f > 0$ and on at least one of the vretexes n_t = -1, this element needs surface integration. If e_t = 2, this element needs line integration.

For the touched problem, additional line elements should be added to improve the accuracy, which exist when surface integration changes to edge integration. By using the flags proposed, it is very convenient to identify the additional line elements, which will be reported in the full paper.

III. EXAMPLES AND RESULTS

The accuracy of the proposed method is verified by a simple example which has analytical solutions. As shown in Fig. 1, each PM is 20mm×10mm×1000m and B_r=1.1 tesla, $\mu=\mu_0$. The length of PM objects along the z direction is 100000 mm. It is about 100000 times the *xy* dimensions so that it is accurate enough using 2-D FEM to represent finite (3-D) magnets. When the distance between them is zero, they touch each other. Table I shows the computed force versus distance between the two objects using 2-D FEM (with 33168 elements) and the analytical method. It shows that the results of the numerical method coincide with that of the analytical method. Table II shows that when the two objects touch each other, when using different scales of finite element mesh, the computed total force and the force contributed from the additional line elements. It indicates the importance of the introduction of the additional line elements, especially if coarse meshes are used. It is noted that if the number of elements of the mesh increases, the force value from additional line elements will reduce. This makes sense because if the mesh is refined, the size of each line element will become small, and the contribution from the integration on it will also become small.

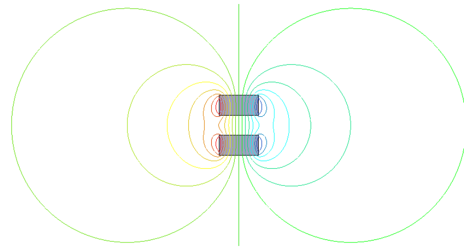


Fig. 1. Flux distribution of two PM objects attracting one another.

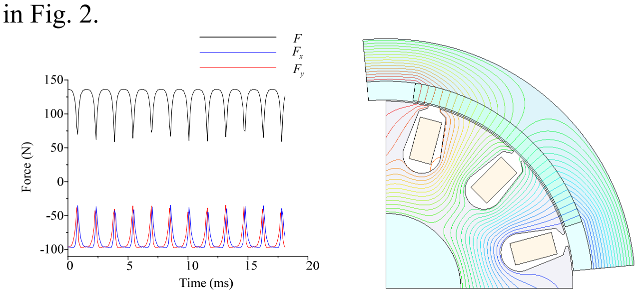
TABLE I
COMPARISON OF FORCES BETWEEN ANALYTICAL AND 2-D FEM RESULT

Gap (m)	Analytical Result (N/m)	Numerical Result (N/m)
0.010	854.20	854.09
0.009	951.30	951.27
0.008	1063.23	1063.1
0.007	1193.16	1193.1
0.006	1345.31	1345.0
0.005	1525.57	1525.2
0.004	1742.54	1742.5
0.003	2009.80	2009.8
0.002	2351.52	2351.6
0.001	2820.78	2820.8
0.000	3678.26	3678.2

TABLE II
THE TOTAL FORCE AND THE FORCE VALUE FROM ADDITIONAL LINE ELEMENTS VERSUS NUMBER OF ELEMENTS

Number of elements of FEM mesh	Total force (N/m)	Force value from additional line elements (N/m)
362	3453.0	327.24
694	3609.8	271.38
1278	3658.9	165.09
3904	3670.9	90.78
8794	3675.0	47.49
16562	3676.6	36.97
33168	3678.2	24.27

The method is further applied to compute the magnetic force distribution and the global force in a surface mounted PM machine in which the PMs touch the iron core, as shown in Fig. 2.



(a) (b)

Fig. 2. Magnetic force calculation. (a)Force waveforms versus time. (b)Magnetic flux distribution.

IV. REFERENCES

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